Advanced Modern Macroeconomics

Capital Policy and Business Cycles

Max Gillman

University of Missouri, St. Louis

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Chapter 6: Capital Policy and Business Cycles

Chapter Summary

- Business Cycle Policy Issue:
  - over business cycle, capital investment fluctuates a lot;
  - in downturn, investment can fall so much as to concern;
  - occasional dramatic investment decreases during crises .
- Long Term Policy Issue:
  - how investment is affected by taxes.
- Chapter shows investment cycles in two period model
  - through simultaneous change in goods productivity
  - and in initial period exogenous income endowment.
- Shows effect of fixed interest rate during downturn
  - gives an "excess supply of capital"
  - policy issue of Keynes 1936 General Theory and today.

Building on the Last Chapters

- Same two-period model of last Chapter 5
- Now applied to business cycle through simple comparative statics.
- Combines 5% increase in income endowment and in productivity.
- Here endowment is current period income
  - rather than time endowment in Chapter 3
- Here is capital productivity
  - rather than labor productivity in Chapter 3.
- Unemployed capital defined with fixed price.
  - similar to unemployed labor in Chapter 3 with fixed wage
- Capital tax analysis parallel to labor tax of Chapter 3.

Learning Objective

- Formulate changes in supply, demand in goods, capital markets,
  - with general equilibrium representation
  - to show features of economic expansion, contraction.
- See concept of surplus capital, and connection to *General theory*.
- Realize that change in productivity not sufficient to generate cycle
  - also necessary to add change in income endowment.
- Identify marginal distortion of taxes on capital
  - even with tax revenue returned lump sum to consumer;
  - how taxes distort investment..

Who Made It Happen

- Keynes 1936 uses capital market to illustrate
  - decrease in interest rate
  - caused by capital supply shifting out by more than capital demand.
  - Seen also in our business cycle model.
- Keynes: interest rate determined in money market, not capital market.
  - Modern analysis echo: "monetary policy sets interest rate" .
  - although monetary policy within realm of capital markets.
- Keynes focused on crisis periods, with excess supply of capital
  - savings, investment not equal over business cycle.
  - in his Treatise of Money 1930, and 1936 General Theory
- Excess savings and "liquidity trap";
  - now called "zero nominal bound" issue.
  - Keynes: increase in capital supply has no effect on real interest rate.
- Happens if capital supply curve horizontal at low interest rate.

Keynes's Policy Prescription

- "Excess savings" not being turned into investment,
- so government should step in and do investment.
- Keynes: "I am now somewhat sceptical of the success of a merely monetary policy directed towards influencing the rate of interest. I expect to see the State ... taking an ever greater responsibility for directly organizing investment."
- Current issue with international banking sector collapse 2007-2009

Business Cycles and Investment

- Business cycles: explain by change in productivity?
- But: productivity increase, marginal product rises, investment unchanged.
- Substitution effect exactly offsets income effect
- Another dimension of RBC puzzle
- Makes harder to understand policy issues of big decrease in investment.
- So add change in current period income endowment.
- Supply of capital shifts out, demand function same.
- Need both so interest rate and investment co-move with output.

Economic Expansion: Example 6.1

$$A_G = 12, y_0 = 100 \rightarrow A_G = 12.6, y_0 = 105$$

$$k_1^d = \frac{36}{(1+r)^2} \rightarrow k_1^d = \frac{39.69}{(1+r)^2}. \quad (1)$$

$$\Pi_1 = 12\left[\frac{36}{(1+r)^2}\right]^{0.5} - \frac{36(1+r)}{(1+r)^2} = \frac{36}{1+r} \quad (2)$$

$$\rightarrow \Pi_1 = 12.6\left[\frac{39.69}{(1+r)^2}\right]^{0.5} - \frac{39.69(1+r)}{(1+r)^2} = \frac{39.69}{1+r}. \quad (3)$$

$$k_1^s = \frac{y_0\beta}{1+\beta} - \frac{\Pi_1}{(1+\beta)(1+r)} = \frac{y_0\beta}{1+\beta} - \left(\frac{1}{1+\beta}\right)\frac{36}{(1+r)^2};$$

$$\rightarrow k_1^s = \frac{105(0.98)}{1.98} - \frac{39.69}{(1.98)(1+r)^2}.$$

Equilibrium in Expansion

$$
\begin{aligned}
r &= \left(\frac{36\,(2.98)}{100\,(0.98)}\right)^{0.5} - 1, \rightarrow \left(\frac{39.69\,(2.98)}{105\,(0.98)}\right)^{0.5} - 1; \\
r &: \quad 0.0463 \longrightarrow 0.0721. \\
k_1^s &= \frac{105\,(0.98)}{1.98} - \frac{39.69}{(1.98)\,(1.0721)^2} = 34.531 = \frac{39.69}{(1.0721)^2} = k_1^d. \\
c_1 &= y_1 = \frac{12.6\,(39.69)^{0.5}}{1.0721} = 74.042;\ \Pi_1 = \frac{39.69}{1.0721} = 37.021. \\
c_0^d &= 105 - k_1^s = 105 - 34.531 = 70.469. \\
y_1 &= c_1^d = \Pi_1 + k_1^s\,(1+r) = 37.021 + 34.531\,(1.0721) = 74.042.
\end{aligned}
$$

Utility, Growth and Capital Supply and Demand Functions

$$u = \ln c_0^d + \beta \ln c_1^d = \ln 70.469 + 0.98 \ln 74.042 = 8.4737,$$

$$c_1 = \left(\frac{e^u}{c_0}\right)^{\frac{1}{\beta}} = \left(\frac{e^{8.4737}}{c_0}\right)^{\frac{1}{0.98}}.$$

$$g = 0.0255 \rightarrow g = \frac{c_1}{c_0} - 1 = \frac{74.042}{70.469} = 0.0507,$$

$$k_1^s = \frac{105\,(0.98)}{1.98} - \frac{39.69}{(1.98)\,(1+r)^2};\; k_1^d = \frac{39.69}{(1+r)^2},$$

$$1 + r = \left(\frac{39.69}{(105\,(0.98) - (1.98)\,k_1^s)}\right)^{0.5};\; 1 + r = \left(\frac{39.69}{k_1^d}\right)^{0.5}.$$

5% Increase in Income Endowment and Productivity Increases Investment and Interest

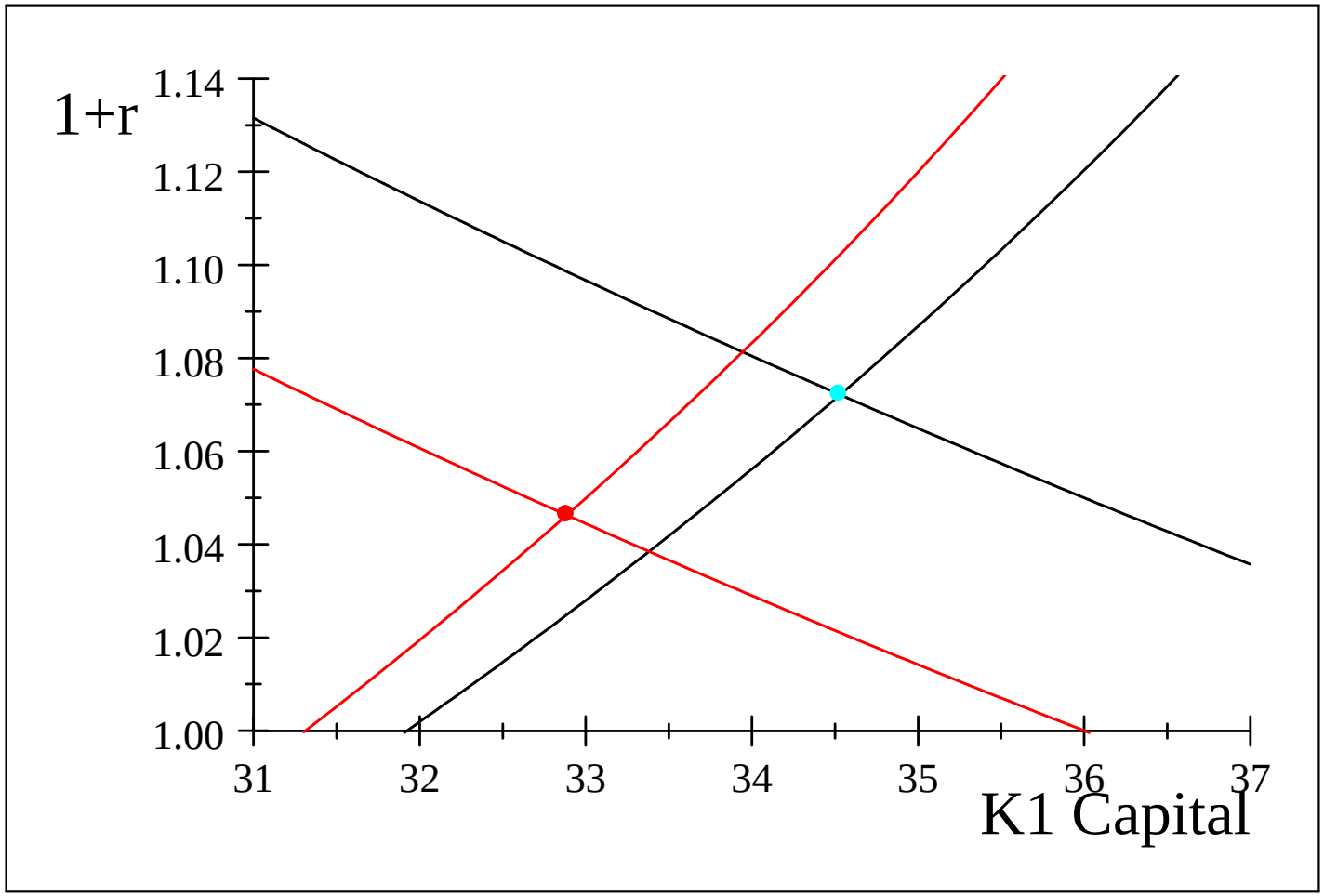

Figure 6.1. Capital Market with 5% Increase in Goods Productivity and Income Endowment (blue) in Example 6.1 versus Example 5.3 (red).

Budget Line, Utility Level Curve and Production Function

$$
\begin{aligned}
c_1^d &= \Pi_1 + k_1^s (1+r) = \frac{39.69}{1.0721} + (105 - c_0)(1.0721); \\
c_1^d &= \left(\frac{e^u}{c_0}\right)^{\frac{1}{\beta}} = \left(\frac{e^{8.4737}}{c_0}\right)^{\frac{1}{0.98}}; \\
c_1^s &= 12.6\sqrt{k_1^d} = 12.6\,(105 - c_0)^{0.5}.
\end{aligned}
$$

Expansion with Steeper Equilibrium Budget Line

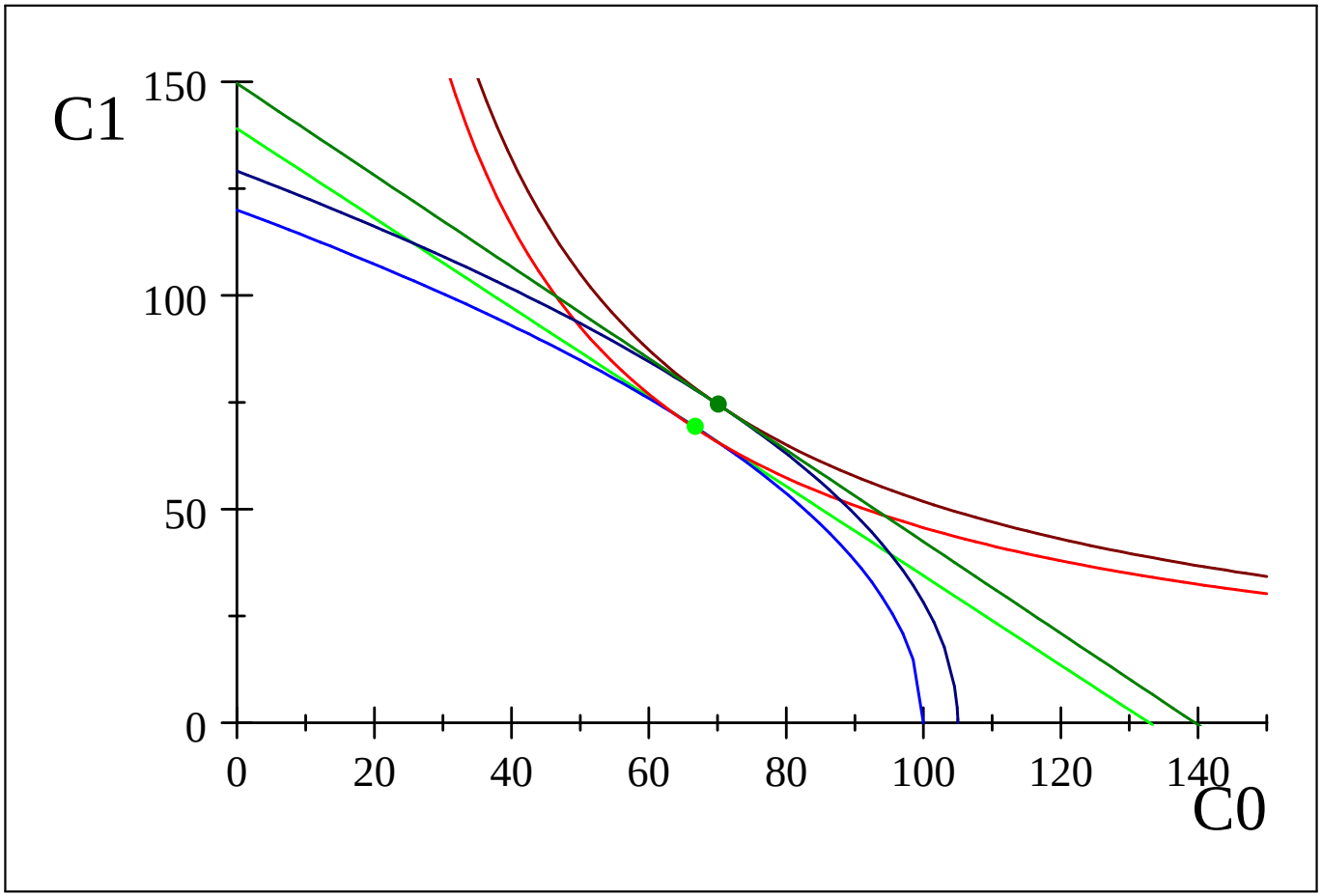

Figure 6.2. Economic Expansion in Example 6.1 (dark red, blue and green) compared to Baseline Example 5.3 (light red, green, blue).

Example 6.2: Contraction

$A_G = 12$ and $y_0 = 100 \longrightarrow \longrightarrow A_G = 11.4$ and $y_0 = 95$.

$$k_1^s = \frac{95\,(0.98)}{1.98} - \frac{\Pi_1}{(1.98)\,(1+r)};\; k_1^d = \left[\frac{(0.5)\,(11.4)}{(1+r)}\right]^2 = \frac{32.49}{(1+r)^2}.$$

$$\Pi_1 = 11.4\left[\frac{[(0.5)\,(11.4)]^2}{(1+r)^2}\right]^{0.5} - \frac{32.49\,(1+r)}{(1+r)^2} = \frac{32.49}{1+r}.$$

$$k_1^s = \frac{95\,(0.98)}{1.98} - \frac{32.49}{(1.98)\,(1+r)^2} = \frac{32.49}{(1+r)^2} = k_1^d.$$

$$\begin{aligned} 1+r &= \left(\frac{32.49\,(2+\beta)}{95\,(\beta)}\right)^{0.5} = \left(\frac{32.49\,(2.98)}{95\,(0.98)}\right)^{0.5} = 1.0198; \\ r &= 0.0463 \longrightarrow r = 0.0198. \end{aligned}$$

Equilibrium Quantities in Contraction

$$k_1^s = \frac{95\,(0.98)}{1.98} - \frac{32.49}{(1.98)\,(1.0198)^2} = 31.24 = \frac{32.49}{(1.0198)^2} = k_1^d.$$

$$y_1 = \frac{11.4\,(32.49)^{0.5}}{1.0198} = 63.72; \tag{4}$$

$$\Pi_1 = \frac{32.49}{1.0198} = 31.86; \tag{5}$$

$$c_0^d = 95 - k_1^s = 95 - 31.24 = 63.76; \tag{6}$$

$$\begin{aligned} y_1 &= c_1^d = \Pi_1 + k_1^s\,(1+r)\,, \\ y_1 &= 31.86 + 31.24\,(1.0198) = 63.72. \end{aligned} \tag{7}$$

Utility, Growth, and Capital Supply and Demand Functions

$$
\begin{aligned}
u &= \ln c_0^d + \beta \ln c_1^d = \ln 63.76 + 0.98 \ln 63.72 = 8.2265, \\
c_1 &= \left(\frac{e^u}{c_0} \right)^{\frac{1}{\beta}} = \left(\frac{e^{8.2265}}{c_0} \right)^{\frac{1}{0.98}}.
\end{aligned}
$$

$$
1 + g = \frac{c_1}{c_0} = \frac{63.72}{63.76} = 0.999, \; g = -0.001
$$

$$
k_1^s = \frac{95\,(0.98)}{1.98} - \frac{32.49}{(1.98)\,(1+r)^2}, \; k_1^d = \frac{32.49}{(1+r)^2}, \tag{8}
$$

$$
1 + r = \left(\frac{32.49}{(95\,(0.98) - (1.98)\,k_1^s)} \right)^{0.5}, \; 1 + r = \left(\frac{32.49}{k_1^d} \right)^{0.5}. \tag{9}
$$

Contractionary Shift Back in Capital Supply and Demand

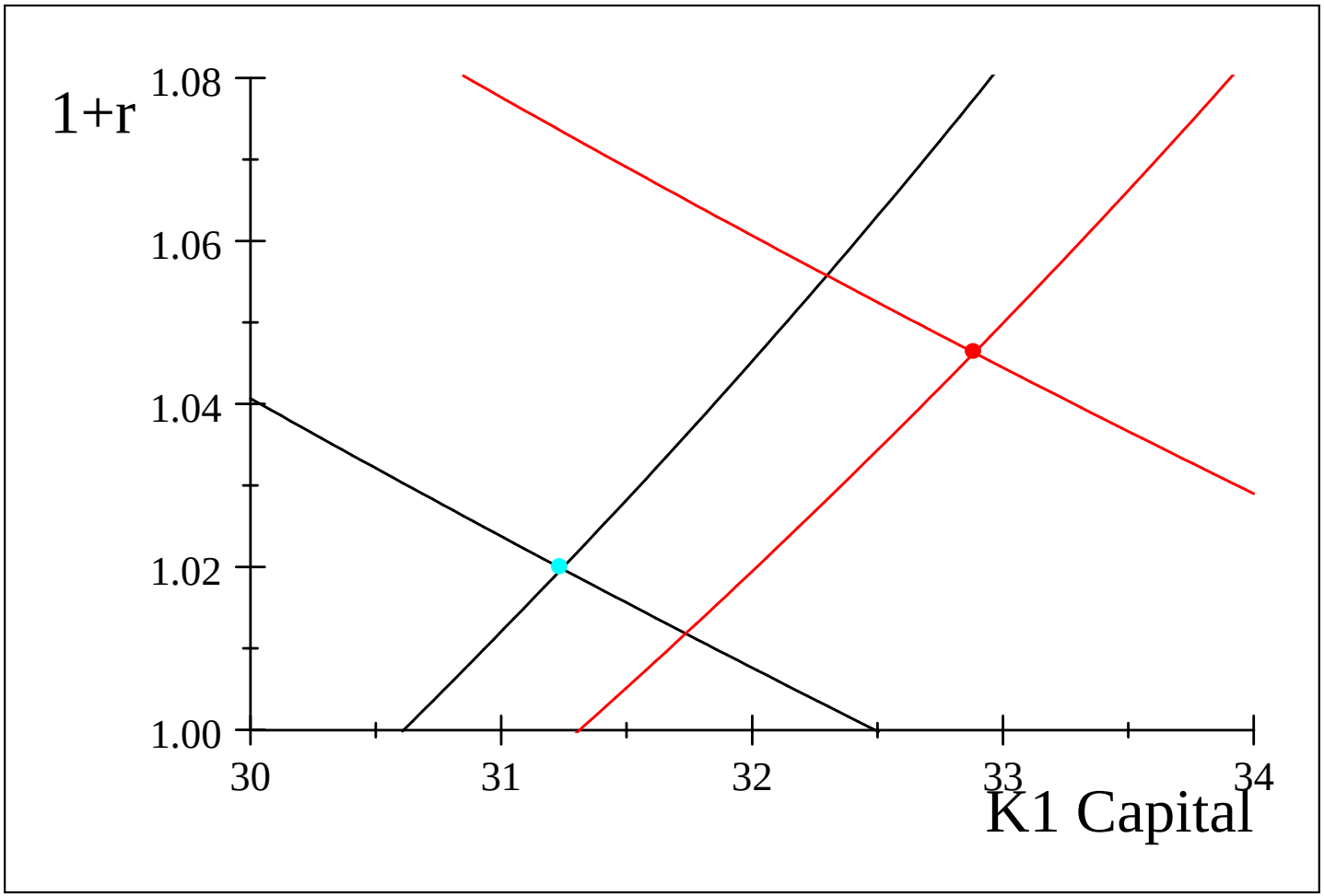

Figure 6.3. Capital Market with 5% Decrease in Goods Productivity and Income Endowment (blue) in Example 6.2 versus Example 5.3 (red).

Budget Line, Utility Level Curve and Production Function in Contraction

$$
\begin{aligned}
c_1^d &= \Pi_1 + k_1^s\left(1+r\right) = \frac{32.49}{1.0198} + \left(95 - c_0\right)\left(1.0198\right); \\
c_1^d &= \left(\frac{e^u}{c_0}\right)^{\frac{1}{\beta}} = \left(\frac{e^{8.2265}}{c_0}\right)^{\frac{1}{0.98}}; \\
c_1^s &= 11.4\sqrt{k_1^d} = 11.4\left(95 - c_0\right)^{0.5}.
\end{aligned}
$$

Flatter Budget Line in Contraction as Interest Rate Decreases

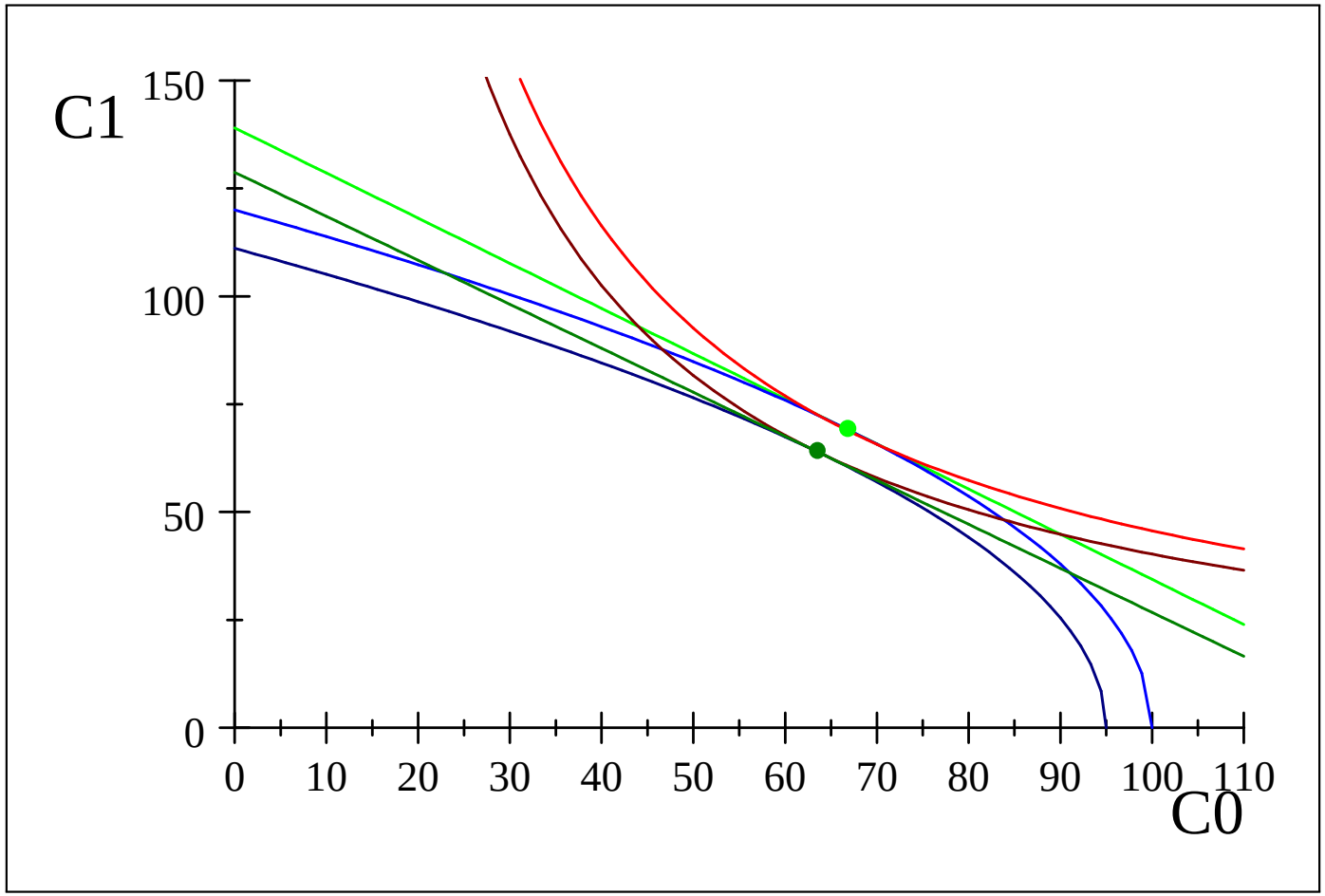

Figure 6.4. Economic Contraction in Example 6.2 (dark red, blue and green) compared to Baseline Example 5.3 (light red, green, blue).

Concept of Unemployed Capital

- Crisis periods may have extreme decrease in investment
- Keynes's (1936) concept of excess, unused, capital.
- Surplus capital if assume price rigidity of interest rate, and recession.
- "Liquidity trap" of Keynes with excess capital supply.
- Samuelson (1951) called such excess supply "depression economics".

Example 6.3 Fixed Price

- $r = \bar{r} = 0.0463$.
- $A_G = 12$, $y_0 = 100$, $\rightarrow A_G = 11.4$, $y_0 = 95$.

$$k_1^d = \frac{32.49}{(1+r)^2}\frac{32.49}{(1.0463)^2} = 29.68,$$

$$k_1^s = \frac{95\,(0.98)}{1.98} - \frac{32.49}{(1.98)\,(1.0463)^2} = 32.03.$$

$$1 + \bar{r} = \left(\frac{\frac{32.49}{(1.98)}}{\left(\frac{95(0.98)}{1.98} - k_1^s\right)} \right)^{0.5}, \tag{10}$$

$$1 + \bar{r} = \left(\frac{32.49}{k_1^d} \right)^{0.5} .. \tag{11}$$

Shift Down in Capital Supply and Demand with Fixed Interest Rate

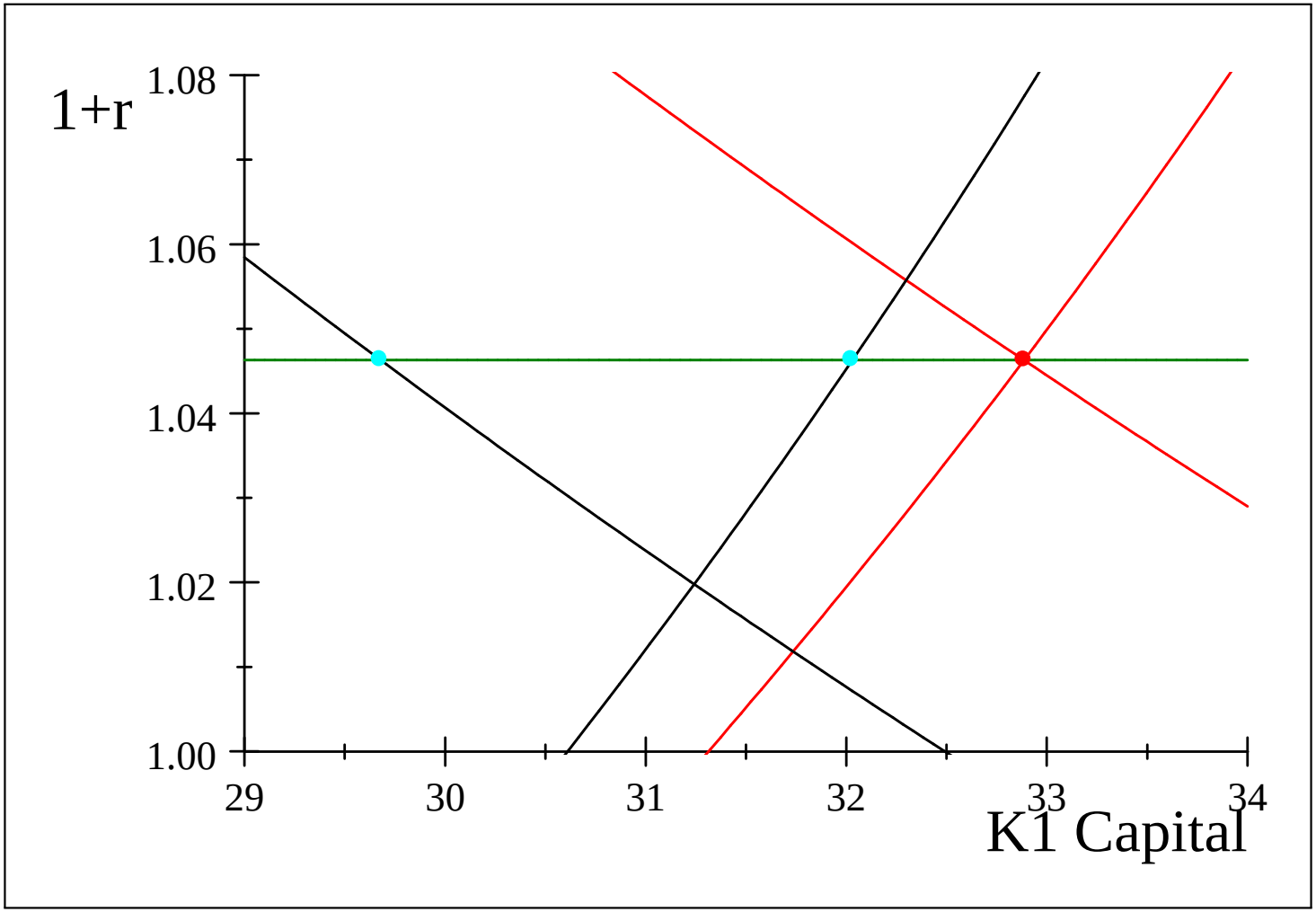

Figure 6.5. Deeper Recession with a Fixed Interest Rate in Example 6.3.

Excess Capital Supply and Goods Demand

- *Excess Capital Supply* : $k_1^s - k_1^d = 32.03 - 29.68 = 2.35$.
- $\frac{2.35}{32.89} = 0.071$, or 7% of capital unemployed.

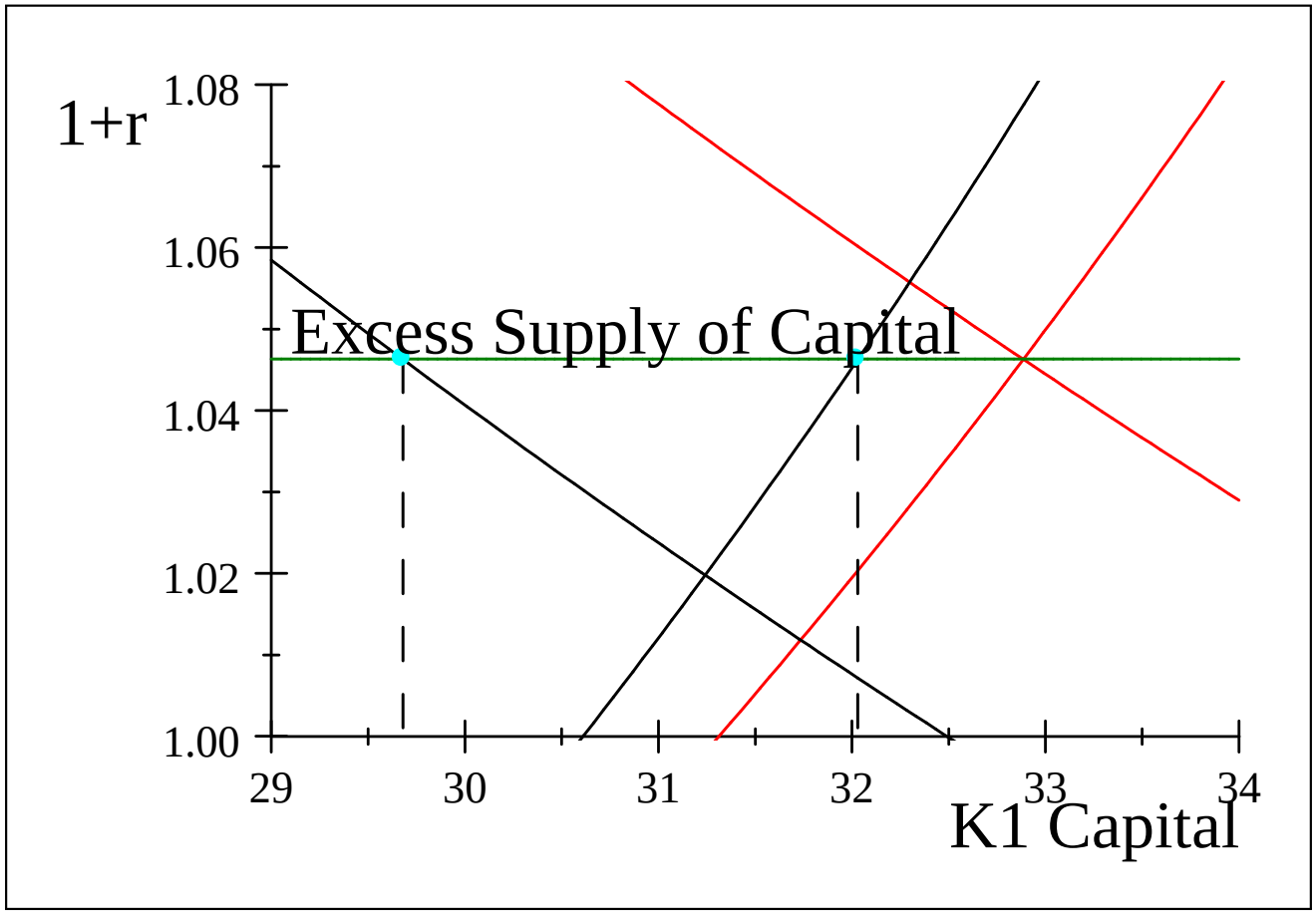

Figure 6.6. Excess Capital Supply Indicated by Horizontal Distance between Dotted Lines in Example 6.3.

Policy for Capital Investment

- Fixed interest rate during recession:
  - big drop capital investmentemployment
  - big drop in future consumption.
- What policy to offset such crisis decreases?
- Tax changes sometime used as policy tools
  - But best used for increasing long term efficiency of government by moving towards lower tax rates over time.

A Tax on Capital Income

- Tax rate τ_k on consumer's capital income rather than firm's.
- Firm problem unchanged
- Budget Constraint: $(1+r)\,k_1\,(1-\tau_k)$ is after capital-tax income
- Tax transfer T goes to consumer:

$$c_1 = \Pi_1 + k_1^s(1+r)\,(1-\tau_k) + T. \tag{12}$$

- Government budget constraint:

$$T = \tau_k\,(1+r)\,k. \tag{13}$$

Example 6.4

$$\begin{aligned}
\underset{k_1^s}{\text{Max}}\ u &= \ln\left(y_0 - k_1^s\right) + \beta \ln\left[\Pi_1 + k_1^s(1+r)\left(1-\tau_k\right) + T\right]. \\
0 &= \frac{-1}{y_0 - k_1^s} + \beta \frac{(1+r)\left(1-\tau_k\right)}{\Pi_1 + k_1^s(1+r)\left(1-\tau_k\right) + T}.
\end{aligned}$$

$$T = \tau_k (1+r) k_1, \rightarrow k_1^s = \frac{y_0 \beta (1+r)\left(1-\tau_k\right) - \Pi_1}{(1+r)\left(1+\beta\left(1-\tau_k\right)\right)}$$

$$y_1 = 12\left(k_1^d\right)^{0.5},\ k_1^d = \frac{36}{(1+r)^2},\ \Pi_1 = \frac{36}{1+r}.$$

$$\Pi_1 = \frac{36}{1+r}; \rightarrow k_1^s = \frac{y_0 \beta (1+r)\left(1-\tau_k\right) - \frac{36}{1+r}}{(1+r)\left(1+\beta\left(1-\tau_k\right)\right)}. \tag{14}$$

Capital Supply and Demand Equations with Tax

$$\tau_k = 0.10$$

$$1 + r = \left(\frac{36}{100\left(0.98\right)\left(1 - 0.1\right) - k_1^s\left(1.98\left(1 - 0.1\right)\right)} \right)^{0.5}; \quad (15)$$

$$1 + r = \frac{36}{\left(k_1^d\right)^2}. \quad (16)$$

$$\begin{aligned} k_1^s &= k_1^d, \Longrightarrow \\ \frac{100\left(0.98\right)\left(1 + r\right)\left(1 - 0.1\right) - \frac{36}{1+r}}{\left(1 + r\right)\left(1.98\left(1 - 0.1\right)\right)} &= \frac{36}{\left(1 + r\right)^2}, \rightarrow r = 0.066 \end{aligned}$$

- Shifts back Capital Supply function.
- Leaves Capital Demand function unchanged.

Capital Tax Raises Interest Rate and Reduces Investment

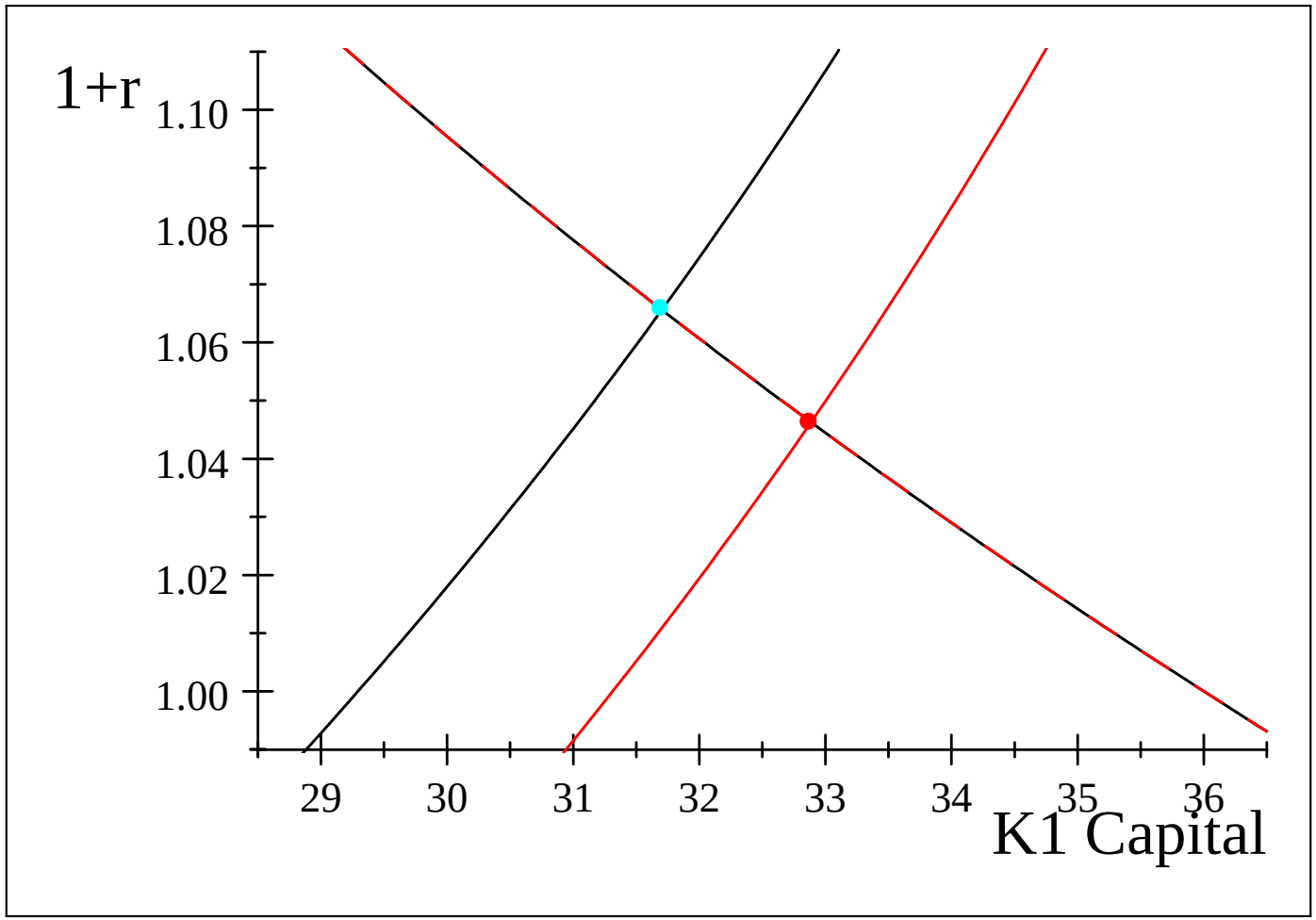

Figure 6.7. The Shift Back in Savings with a 10% Tax in Example 6.4.

Market for Future Consumption

$$c_1^s = 12\sqrt{k_1^d} = \frac{72}{1+r},\ 1+r = \frac{72}{c_1^s}.$$

$$\begin{aligned} c_1^d &= \Pi + k_1^s(1+r) = \frac{36}{1+r} + k_1^s(1+r), \\ 0 &= 15.798\left(\frac{1}{1+r}\right)^2 - c_1^d\left(\frac{1}{1+r}\right) + 49.495. \end{aligned}$$

$$\begin{aligned} \frac{1}{1+r} &= \frac{-B-\sqrt{B^2-4AC}}{2A} \\ &= \frac{c_1^d - \sqrt{\left(c_1^d\right)^2 - 4(15.798)(49.495)}}{2(15.798)}. \end{aligned}$$

- c_1 Supply Unchanged; Demand shifts back.

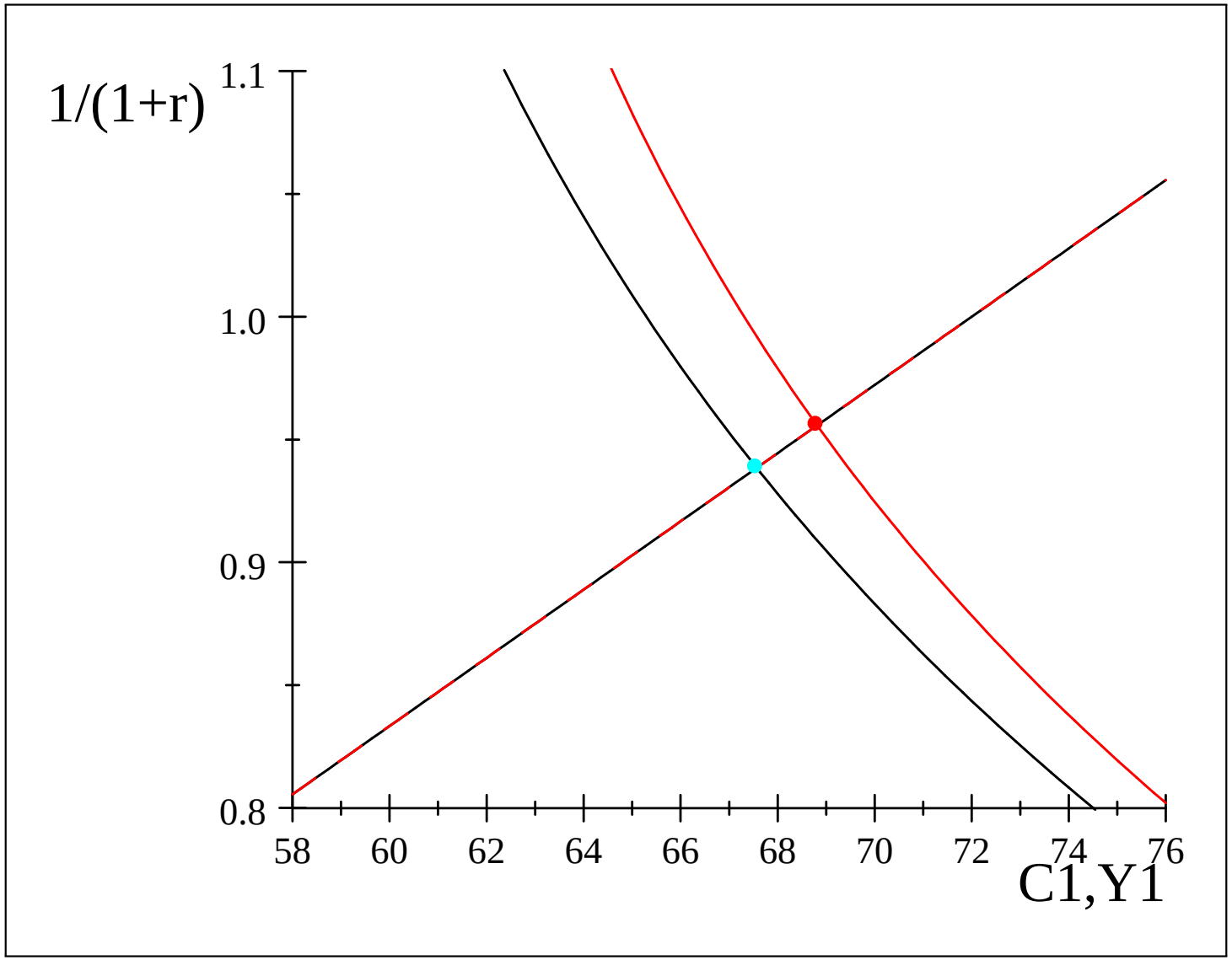

Figure 6.8. Future Period Consumption with 10% Tax in Example 6.4.

Utility Level Curve, Production Function, Budget Line

- $$\begin{aligned} u &= \ln\left(100 - k_1^s\right) + 0.98 \ln\left(c_1^d\right), \\ u &= \ln(100 - 31.704) + 0.98 \ln(67.568) = 8.3527 \\ c_1^d &= \left(\frac{e^u}{c_0}\right)^{\frac{1}{\beta}} = \left(\frac{e^{8.3706}}{c_0}\right)^{\frac{1}{0.98}}. \end{aligned}$$

$$c_1^s = 12\sqrt{k_1^d} = 12\left(100 - c_0\right)^{0.5},$$

$$c_1^d = \Pi_1 + k_1^s\left(1 + r\right) = \frac{36}{1.0656} + \left(100 - c_0\right)(1.0656).$$

- Utility level falls; tax wedge exists; budget line intersects production function rather than being tangent; utility level curve remains tangent to budget line.

Capital Policy Alternatives

- Fixed interest rate a shortcut to explain crisis
- Better to explain what is wrong in capital markets:
  - bank industry for example failed in 1930s and 2007-2009.
  - More a bank insurance failure than fixity of interest rates.
  - Unexpected changes to banking productivity: de facto aggregate risk.
- Internalize such risk into markets as aim of policy solutions.
- Alternative: borrow from future generations and spend now: Keynes
- Markets build in expectations of future taxes:
  - either implicit inflation tax or explicit taxes.

Capital Market Regulation: Usury and Economic Science

- 325 AD: Roman Emperor Constantine I prohibited interest on loans
- Known as usury: not just excessive interest, but at times all interest.
- 1215: Article 10 of British Magna Carta.
  - its usury prohibition was repealed in 1297 version.
- 14th century: Pope Clement V deems interest heresy.
- 1745: Pope Benedict XIV forbids charging interest.
- 1776: Adam Smith *Wealth of Nations:* allow unregulated capital interest
  - "Notwithstanding the Edict of 1766, by which the French king attempted to reduce the rate of interest from five to four percent, money continued to be lent in France at five percent, the law being evaded in several different ways." (p.380 Book II Chapter 4).
- Gave birth to modern economics in part
- "Western" law versus Islamic banking that continues interest prohibition: a fight over law of Interest?